Exercise 26

Find the gradient vector field ∇f of f and sketch it.

$$f(x, y) = \frac{1}{2}(x^2 - y^2)$$

Solution

Calculate the gradient and call it **F**.

$$\mathbf{F} = \nabla f$$

$$= \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y} \right\rangle f$$

$$= \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle$$

$$= \left\langle \frac{\partial}{\partial x} \left[\frac{1}{2}(x^2 - y^2) \right], \frac{\partial}{\partial y} \left[\frac{1}{2}(x^2 - y^2) \right] \right\rangle$$

$$= \left\langle \left[\frac{1}{2}(2x) \right], \left[\frac{1}{2}(-2y) \right] \right\rangle$$

$$= \langle x, -y \rangle$$

The vector field of this gradient is superimposed on a contour plot of $f(x, y)$. Notice that the vectors are perpendicular to each of the contours, pointing in the direction of greatest increase.